

STUDY OF MOTION CONTROL OF A FLEXIBLE LINK

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By

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CERTIFICATE

This is to certify that the thesis entitled “**STUDY OF MOTION CONTROL OF A FLEXIBLE LINK**” submitted by **Swadheen Satyakam Mishra** bearing roll no **111ME0312** in partial fulfilment for the award of the degree of **Bachelor of Technology** in **Mechanical Engineering** at National Institute of Technology, Rourkela is an authentic work carried out under my supervision and guidance. To the best of my knowledge, the matter embodied in this report has not been submitted to any other University/Institute for the award of any Degree or Diploma.

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Contents

Abstract	5
Chapter 1: Introduction.....	6
1.1 Automation, Robotics and Mechatronics	6
1.2 Single Link Flexible Manipulators	6
1.3 Applications.....	7
1.4 Advantages	7
1.5 Disadvantages	8
Chapter 2: Literature Review	9
Chapter 3: Modelling.....	16
3.1 Problem Formulation	16
3.2 Model.....	16
3.3 Simulation.....	Error! Bookmark not defined.
Chapter 4: Results.....	21
Chapter 5: Conclusion	29
5.1 Summary.....	29
5.2 Future Scope Of Work.....	29
References	30

ABSTRACT

20th century has witnessed massive upsurge in the use of manipulators in several industries especially in space, defense, and medical industries. Among the types of manipulators used, single link manipulators are the most widely used. A single link robotic manipulator is nothing but a link controlled by an actuator to carry out a particular function such as placing a payload from point A to point B. For low power requirements single link manipulators are made up of lightweight materials which require flexibility considerations. Flexibility makes the dynamics of the link heavily non-linear which induces vibrations and overshoot. In this project initially the dynamic model of rigid flexible manipulator is explained, then the state space model of the manipulator system is incorporated into MATLAB. The link flexibility is studied by a single beam FE model, where expressions for kinetic and potential energy are employed to derive the torque equation. The 3 flexible link equations are coupled in terms of 3 variables, θ , ϕ and v . The tip angle is finally given as $\left(\theta + \frac{v}{l}\right)$ for flexible case whereas for the rigid manipulator the tip angle is same as the hub angle θ . Therefore accurate computation of v is very important. The joint flexibility is excluded from analysis. Several comparisons were made between the rigid and flexible link for torque requirement. The relation between the trajectory and hub angle is also plotted in a graph. Finally a PD controller taking the errors and its derivative is designed based on the rigid link dynamics.

CHAPTER 1: INTRODUCTION

1.1 AUTOMATION, ROBOTICS AND MECHATRONICS

Industrial revolution necessitated faster and automated processes which towards the onset of the twentieth century, gave rise to the focused discipline of robotics and mechatronics concerning with systems that automate processes using intelligence and strategies stored onto those devices or machines. Following the wide variety of tasks the robots are supposed to carry out; the way they are designed varies greatly. Some robots have multiple mechanical links in order to implement a complex trajectory motion while some others have a relatively simpler trajectory owing to the fact that the task these are intended to perform are relatively simpler in terms of the kinematics and dynamics involved in the trajectory such as a circular trajectory. When the motion path is a simple one the number and type of kinematic links required in a body of robot greatly reduces. There are many ways in which just one or two links are arranged in such a way so as to act as a robotic system. Most often it reduces to just one mechanical link; which rotates around a central hub, which is often a motor; that acts as a robotic manipulator when coupled with an end effector which is determined according to the specific task the robot is supposed to carry out.

1.2 SINGLE LINK FLEXIBLE MANIPULATORS

A single link flexible manipulator is intended for automation with only one mechanical link which is elastically flexible. Theoretically the mechanical links are rigid and flexible without any damping effect of air or its properties that contribute to damping and gradual slowing down of the undesirable vibrations. Single link manipulators are robots with single mechanical link for all the kinematic constraint satisfaction. Generally the link is attached to a central hub on a rotating joint that houses the motor on the

shaft of which the link is mounted upon. The other end of the link usually houses an end effector which is according to the specific task the robot is to perform. It usually have a payload mass to carry from point A to point B in its path.

1.3 APPLICATIONS

Now-a-days the applications of robotic systems with a single link flexible manipulator is on a constant rise. Industries such as space, automobile, health, defense etc., have started implementing flexible manipulators in order to perform industry-specific tasks more efficiently and effectively. Several crucial applications such as vibration reduction in the remote manipulator system of a space shuttle while assembling a space station , placing the nozzle dam and decontamination of a nuclear steam generator , cleaning and remediating waste tank, application in microsurgical environments , placing a patient system for cancer treatment , de-burring , surface polishing , and grinding, painting, and drawing applications and biped-walking and walking machines.

1.4 ADVANTAGES

Reduction in the weight of the arms due to less number of powerful actuators attached, greater ratio of weight of payload to that of robot increasing the payload capacity, more access and reach due to long and slender construction, flexibility, decreased damages because of the low inertia, cheaper construction due to fewer link materials, increased acceleration due to less weight of links are several of the advantages of these manipulators over the more traditional manipulators. Large work-space volume is another prominent

advantage these manipulators have over their traditional counterparts. Most importantly these advantages consequentially help in reducing power usage.

1.5 DISADVANTAGES

One of the most important problem these kind of manipulators face is its flexibility and the associated problems. Vibrations due to structural flexibility leads to overshoot and residual undesirable vibrations. Static deflections also arise from vibrations. The highly non-linear vibration decrease end point accuracy and increase settling time or damping time. So accurate modelling is required that includes the bending deflection of the links. Accordingly, the control system is designed so that the tip can follow a trajectory.

In the present work, dynamics of a single link rigid and flexible manipulators are studied. The single beam FE model is used to derive 3 equation of motions in terms of the 3 variables, θ (hub angle), ϕ and v (tip deflection and slope). Two trajectories (at the hub) of input motion are considered and static deflection and slope were used to predict the torques. Also if the torque is given, by solving the differential equation, the hub angle variation is plotted. Lastly a PD controller approach is implemented. Chapter 2 gives literature review of the old work. Chapter 3 provides the model. Chapter 4 summaries the results and chapter 5 concludes the work.

CHAPTER 2: LITERATURE REVIEW

In order to control the flexible manipulators several control strategies have been developed. But chiefly all of these strategies and principles are divided into several broader categories. The most important criterion through which these are divided is the nature of the control structure i.e. whether the loop is open or closed. Open control strategies do not have any feedback mechanism which makes them less advantageous in situations where it is imperative to keep track of the output and take a feedback in order to improve it. Closed loop structures are closed in a sense that they do have the feedback mechanism which lets the controller know what the output is and what is the error between the desired and the actual output. The feedback mechanism is again divided into two principal ways i.e. feed forward and feed backward. They differ in the direction of the feedback signal being sent into the system or out of it. Active and passive control structures differ in the way they exert the control on the system. Active controls continuously interact with the system and keeps changing the excitation force according to the feedback. But passive control mechanisms usually implement an independent controller which do not interact with the system at all through any sensor and actuator. Adaptive or iterative and fixed strategies differ from each other in the sense that the former changes the algorithm by changing some crucial fixed parameters of the algorithm while the later does not change those parameters.

Inverse dynamic trajectory of a single link flexible manipulator was addressed in [1] . The inverse dynamic equations were solved in the time domain. Separating the inverse system into causal and anti-causal part, it calculated the feed- forward torque and the trajectories of all state variables that do not excite structural vibrations for a given end-point trajectory.

A collocated position controller based upon of a PD feedback control technique was developed in [2] & a command-filter vibration controller was designed inside the position control loop based on the dominant vibration modes of the system. The position controller moved the end-point of the manipulator to a

required position and the vibration controller dampened motion-induced vibrations of the manipulator due to elastic flexibility of the system during fast maneuvers. Low-pass and band-stop elliptic filters were used in the vibration controller to filter out input energy at dominant vibration modes of the manipulator in order to prevent it from excitation at the natural frequencies.

High speed digital to analog convertor components on an electrical interface board along with National Instrument's LABVIEW software package was provided in [3] . Dynamic modelling of the system was on Gibbs-Appell formulation and Assumed Mode Method. An experimental setup of a single-link elastic manipulator was installed. The electrical interface board worked between this experimental setup, user and computer. Input data for the electrical interface board were the number of mode shapes up to 4 and the profile of the input torque via time. The data through the electrical interconnection are transferred to the manipulator system.

A cascade style fuzzy logic controller was used to remove vibrations and obtain fast trajectory tracking performance in [4]. The particular FLC structure consists of 3 different FLCs. Input of the 1st & the 2nd FLCs were motor rotation angle error, its derivative & the end-point deflection error along with its derivative, respectively. Outputs of these were inputs for the 3rd FLC, which sends control signal to the flexible robot arm. Each of the FLCs were embedded in a DS1103 real-time board. Parameters of the flexible arm such as link length & spring stiffness were altered to measure the robustness of the FLC. It was seen that the FLCs were very robust to internal and external disturbances. Considering the results of the experiments, the proposed FLC structure shows efficient control performance in flexible robot arms.

This paper in [5] developed a switching controller for a single flexible link manipulator and provided experimental results for a motion from unconstrained to constrained space. The effect of different levels of positive acceleration feedback on the force response, after a collision with an unexpected object occurs, was examined. It was concluded that small amounts of positive acceleration feedback reduces the force overshoot caused by the collision but that too much can cause instability.

In [6] the modelling of such a robotic arm with a rigid guide and a flexible extensible link subject to quick stops after each complete revolution was considered and its dynamical behavior analyzed. The extensible link that rotates with constant angular velocity has one end constrained to a predefined trajectory. The constrained trajectory allows trajectory control and obstacle avoidance for the active end of the robotic arm.

A mathematical model was derived in [7] using Hamilton's principle. A new variable was introduced before the modal expansion to convert the non-homogeneous boundary conditions into a homogeneous one. The static tip deflection of the flexible link was allowed to maintain the contact force between the end effector and the constrained path and considered in both the inverse kinematics and the order reduction procedures. The state vector of the controller consisted of joint angle of the rigid link, its derivative and integral, the first deflection mode and its derivative, and the integral of contact force. A multivariable controller was designed for the simultaneous motion and force control of the manipulator. The controller consisted of a feedforward term which contributed the torque for the expected joint angles and the contact force, and a feedback term with the time varying optimal gains obtained from the Matrix Riccati equation.

In [8] a modelling of flexible single-link manipulator system using differential evolution and particle swarm optimization technique was presented. The I/O data of the system were acquired through the simulation using finite element method (FDM) based on Lagrangian approach. A bang-bang torque was applied and the responses were studied. A model was chosen and optimized using DE and PSO. One Step Ahead (OSA) prediction, correlation tests and mean squared error (MSE) had been performed. An unseen data was used to test the prediction ability of the model. Results demonstrated the advantages of DE over PSO in parametric modelling of the flexible manipulator system used in this study.

Control strategies with an adaptive long range predictive control algorithm based on the Generalized Predictive Control (GPC) algorithm were developed in [9] and used to address the problem of controlling the contact force exerted by the tip of a structurally flexible manipulator link on its environment. A

simulation with a beam-type continuum and a lumped-parameter model, of the link and of the surface dynamics in the region of contact of a deformable surface respectively was used to test the control strategies. The controller was adaptive in the sense that the system model was identified on-line using a Recursive Least Squares (RLS) algorithm.

New and neuro-fuzzy approaches to tip control of a flexible-link manipulator were presented in [10]. A non-collocated, proportional-derivative (PD) type, fuzzy logic controller (FLC) was developed which performed better than typical model-based controllers (LQR and PD). An adaptive neuro-fuzzy controller (NFC) was developed for situations with payload variability which tunes the I/O scale parameters of the fuzzy controller on-line. The efficacy of the NFC had been evaluated by comparing it with a fuzzy model reference adaptive controller (FMRC).

A manipulator with piezoelectric strips had been considered in [11]. PID controller had been applied for hub angle regulation and H_∞ controller was applied at the piezoelectric actuator to reduce the tip vibrations.

An iterative learning control was proposed in [12] for precise tracking control and end-point vibration suppression. The learning was done in a feedback configuration with hybrid control and the learning law updated the feedforward input from the error of the previous trial. Initially, a collocated proportional-derivative (PD) controller utilizing hub-angle and -velocity feedback was developed. The controller was then extended to incorporate a non-collocated proportional-integral-derivative (PID) controller and a feedforward controller based on input shaping techniques.

This paper [13] considered improving the tip regulation performance of a joint-PD controlled single-link flexible manipulator by introducing nonlinear strain feedback. The controller was developed by applying Lyapunov's direct method. The stability of the closed-loop system was theoretically proven based on the partial differential equations (PDE's) which govern the motion of the flexible robot, instead of using the

traditional truncated models. The controller was very simple in its form, and only the measurements of joint angle, joint velocity, and strain of the bending beam were needed for implementation. The controller was very robust as well because it is independent of system parameters.

This paper [14] presented a new method for simultaneous motion and vibration control of a two-link flexible manipulator using H_2/H_∞ control. A multi-element finite element model of the manipulator was casted into the generalized plant model, and H_2 and H_∞ controllers were synthesized employing the LMI-based optimization algorithm of MATLAB. Finally, a mixed sensitivity H_2/H_∞ control was proposed based on an H_2 norm constrained H_∞ control design.

The manipulator was modelled including gravity terms in [15], and it was verified experimentally that the horizontal and the vertical motions were decoupled for a cylindrically symmetrical link and payload. The mathematical model was used to design a mixed-sensitivity H_2 controller. The sensitivity weighting function was used to obtain the required disturbance rejection properties, including zero sensitivity to a force disturbance in the steady state (i.e. integral action control). The control sensitivity weighting function was chosen to guarantee stability despite variation in the payload mass.

A constrained planar single-link flexible manipulator is dynamically modelled using the assumed mode method in [16]. For testing the effectiveness of the controllers, a Linear Quadratic Regulator (LQR) was developed which was then extended to incorporate a non-collocated PID controller and a feedforward controller based on input shaping techniques. For feedforward controller, positive and modified specified negative amplitude (SNA) input shapers were proposed.

An investigation into simulation and optimization based on genetic algorithm (GA) of a single-link flexible manipulator system in vertical plane was presented by [17]. The model was derived and discretized using the Lagrange equation and using the finite difference (FD) method respectively. GA optimization was implemented to optimize the parameters of PID-based controllers.

Development of an interactive, efficient, user-friendly and easy to use environment for simulation and control of flexible manipulators was presented by [18]. A constrained planar single-link flexible manipulator was considered. A simulation algorithm characterizing the dynamic behavior of the manipulator was developed using finite difference methods. Several open-loop and closed-loop control strategies were developed and incorporated into the environment. The environment was implemented using MATLAB and SIMULINK.

An optimal PID controller for control of vibration flexible manipulator structures using particle swarm optimization for fine-tuning the PID parameters was presented by [19]. The flexible manipulator system was first modelled using finite difference (FD) approach and identified using global search of PSO. The control structure consisted of a combination of conventional PID controller with an intelligent PID-incorporated-PSO controller for position & vibration control. It was noted that proposed controller was effective to move the flexible link to the desired position with suppression of vibration at end-point of a flexible manipulator structure.

A hybrid control scheme consisting of a resonant controller and a fuzzy logic controller (FLC) was used by [20]. The resonant controller was used as the inner loop feedback controller for vibration control using the resonant frequencies at different resonant modes of the system which were determined from experiment. The fuzzy logic controller was designed as the outer loop feedback controller for the tracking control and to achieve zero steady state error.

It presented investigations into the dynamic characterisation of a flexible manipulator system based on finite element methods incorporating structural damping [21]. Performance of the algorithm in describing the dynamic behaviour of the system was assessed in comparison to an experimental test-rig in the time and frequency domains.

Two control structures - fuzzy logic controller and pole-placement controller were designed and compared by [22]. The fuzzy logic control scheme had the hub joint angle error and its derivative as the input of the controller which dampen the joint. Another model based on pole placement control scheme was proposed to place the system pole at a desired location.

CHAPTER 3: MODELLING

3.1 PROBLEM FORMULATION

The problem statement is to design a controller to suppress and regulate the vibration of a single flexible link rotating around a hub attached with a DC – motor. The objective is to rotate the link from point A to point B with minimum vibrations with safest frequency possible and lowest amplitude with a very fast dampening rate preventing overshoot.

3.2 MODEL

Rigid manipulator dynamics

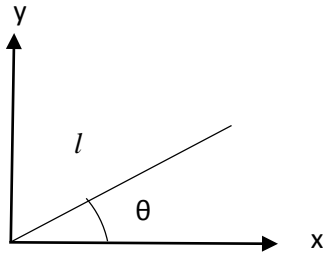


Fig.1 Single Link Rigid Manipulator

Figure.1 shows a single link rigid manipulator. Where l is the length of the link and θ is the angle. The total kinetic energy due to angular motion $\dot{\theta}$ and the linear motion \dot{r} is

$$\text{Mass Factor, } M = \frac{1}{3}ml^2$$

$$\text{Coriolis Factor, } H = 0$$

$$\text{Gravity Factor, } G = \frac{1}{2}mgl\cos\theta$$

$$\text{Torque, } \tau = \frac{1}{3}ml^2\ddot{\theta} + \frac{1}{2}mgl\cos\theta$$

The above equation can also be written as

$$\tau = M(\theta)\ddot{\theta} + C(\theta, \dot{\theta}) + G(\theta)$$

Having the system equation, we have

$$\ddot{\theta} = M(\theta)^{-1}[-C(\theta, \dot{\theta}) - G(\theta)] + \tau$$

Where $M(\theta) = \frac{1}{3}ml^2$ which is the inertia matrix and $G(\theta) = \frac{1}{2}mgl\cos\theta$ is the gravity vector.

Flexible manipulator dynamics

Total displacement vector of an element i is sum of displacement at O_i and deflection of link in local coordinate $O_iQ_iP_i$ as shown in the figure below. Fig 1 shows flexible manipulator.

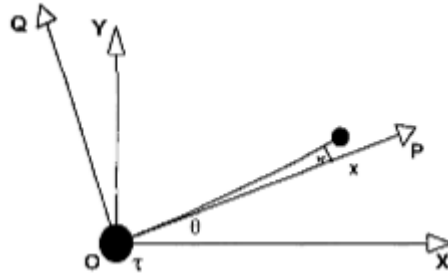


Figure 2 Model of the flexible Link

Mathematically,

$$r_i = r_{oi} + T_i'[l_i + x_i + y_i] \quad \dots (\epsilon 5)$$

Where y_i is deflection due to flexibility in local co – ordinates given by

$$y_i(x_i, t) = N_1(x). V_1 + N_2(x). \emptyset_1 + N_3(x). V_2 + N_4(x). \emptyset_2 \quad \dots (\epsilon 6)$$

Where N_i s are Hermitian shape function and V and θ are bending deflection and slope.

For single link case we write

$$T_i' = \begin{bmatrix} \cos\theta_i & -\sin\theta_i \\ \sin\theta_i & \cos\theta_i \end{bmatrix}$$

Total displacement vector at any point r_i

$$r_i = x \cos \theta_i - \sin \theta_i (N_3 V_2 + N_4 \theta_2) \quad \dots(\epsilon 8)$$

$$x \sin \theta_i - \cos \theta_i (N_3 V_2 + N_4 \theta_2)$$

Assuming $v_i = \theta_i = 0$

$$\text{Kinetic Energy, } KE = T = \frac{1}{2} \int_0^l m (\partial r / \partial t)' (\partial r / \partial t) dx \text{ for particle } i. \quad \dots (\epsilon 9)$$

$$T = \frac{m_i x l_i}{420} [2l_i^2 \dot{\theta}_1^2 \phi_2^2 + 147l_i \dot{\theta}_1 \dot{v}_2 - 21l_i^2 \dot{\theta}_1 \dot{\phi}_2 + 78\dot{\theta}_1^2 v_2^2 + 2l_i^2 \dot{\phi}_2^2 - 22l_i \dot{\theta}_1^2 v_2 \phi_2 + 78\dot{v}_2^2 + 70l_i^2 \dot{\theta}_1^2 - 22l_i \dot{v}_2 \dot{\phi}_2] \dots (\epsilon 10)$$

Potential Energy, $V = \text{Strain Energy} + \text{Gravitational Energy (negligible)}$

$$V = \frac{1}{2} \int E.I. (\partial^2 y / \partial x^2) dx \quad (\epsilon 11)$$

$$V = \frac{2.E.I}{l_i^3} [3v_2^2 - 3l_i v_2 \phi_2 + l_i^2 \phi_2^2] \quad (\epsilon 12)$$

By defining Lagrangian $L = T - V$ and applying Lagrangian principle

$$\partial / \partial t (\partial L / \partial q') - \partial L / \partial q = Q \quad (\epsilon 13)$$

$$Q = \begin{bmatrix} T(t) \\ 0 \\ 0 \end{bmatrix}$$

Putting $q = \theta_1, \dot{\theta}_2, V_2$ and replacing $L = T - V$ in the above equation, we get the following 3 equations

$$\frac{\partial}{\partial t}(\frac{\partial T}{\partial \dot{\theta}_1}) - \frac{\partial T}{\partial \theta} - \frac{\partial V}{\partial \theta_1} = T(t) \text{ (Torque) ---- (€14)}$$

$$\frac{\partial}{\partial t}(\frac{\partial T}{\partial \dot{V}_2}) - \frac{\partial T}{\partial V_2} - \frac{\partial V}{\partial V_2} = 0 \text{ ----- (€15)}$$

$$\frac{\partial}{\partial t}(\frac{\partial T}{\partial \dot{\Phi}_2}) - \frac{\partial T}{\partial \Phi_2} - \frac{\partial V}{\partial \Phi_2} = 0 \text{ ----- (€16)}$$

From equations €14, €15 and €16 coefficients of the terms double and single derivatives of $\theta_1, V_2, \dot{\theta}_2$ are separated and put into the matrix equation of

$$[M]_{3 \times 3} \ddot{Q}' + [C]_{3 \times 3} \dot{Q}'_{3 \times 1} = Q = \begin{bmatrix} T(t) \\ 0 \\ 0 \end{bmatrix}_{(3 \times 1)} \quad (\text{€17})$$

The above matrix representation is pushed into MATLAB code for the representation of the system.

PD Controller:

PD controllers and its variants are the most widely used controllers in industrial loops. It's due to its relatively simpler design, easy implementation, robust nature and very few numbers of tuning parameters.

The control signal provided by PD controller is governed by 2 terms i.e. one proportional and derivatives.

Considering the non-linear equation from the Lagrangian formulation the control input variable τ , represents the torque applied to the link. However, it requires a control in the torque applied at the joint to reach the desired position. Therefore here we considered conventional linear PD law as shown below.

$$\tau = K_p e + K_d \dot{e}$$

τ is the control signal , $e = \theta_d - \theta$, $\dot{e} = \dot{\theta}_d - \dot{\theta}$ is the error signal which is the difference between the reference signal $r(t)$ and output $y(t)$. θ_d is the desired angle and $\dot{\theta}_d$ is the desired velocity. K_p & K_d are the proportional gain and the derivative gain respectively. These parameters are to be tuned to control the system.

The closed loop equation is obtained by substituting the above control law in equation of motion which is reduced into the following form.

$$M(\theta)\ddot{\theta} + C(\theta, \dot{\theta}) + G(\theta) = K_p e + K_d \dot{e}$$

CHAPTER 4: RESULTS

The simulation results of single link rigid and flexible arm inverse dynamics analysis has been done for a joint space trajectory. The geometric parameters considered rigid and flexible links are given in the following table.

Table 1. Geometric Parameters

M_1 (Mass of the system)	5kg
L (length of the link)	1 meter
I (Moment Of Inertia)	$5 \times 10^{-9} \text{ kg m}^2$
E(Young's Modulus)	$20 \times 10^9 \text{ Pa}$
g(Gravitational acceleration)	9.81 m/s^2

- 1) Trajectory in XY plane is defined as follows: $q = \theta \times t$, where $\theta = 30$ degrees and $t \in (0,1)$ seconds.

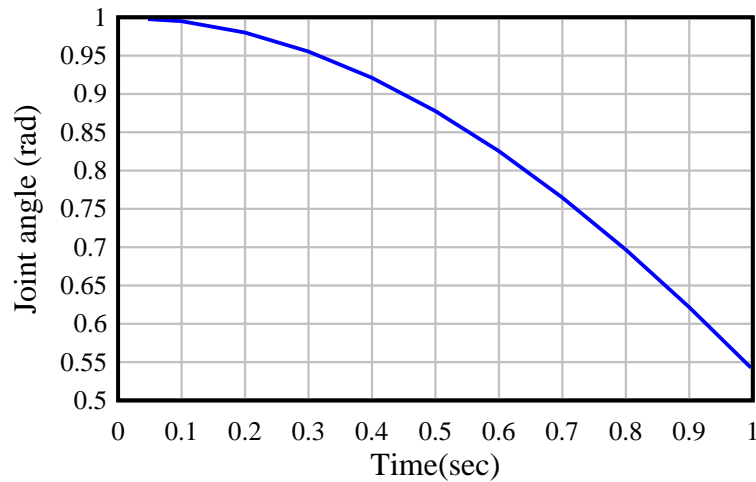


Figure 4 Trajectory

Using inverse dynamic analysis for the above trajectory the corresponding joint torques of rigid and flexible links are shown in the following figures 4 and 5 respectively. Here we can observe that the required joint torques of the rigid link are more than flexible link.

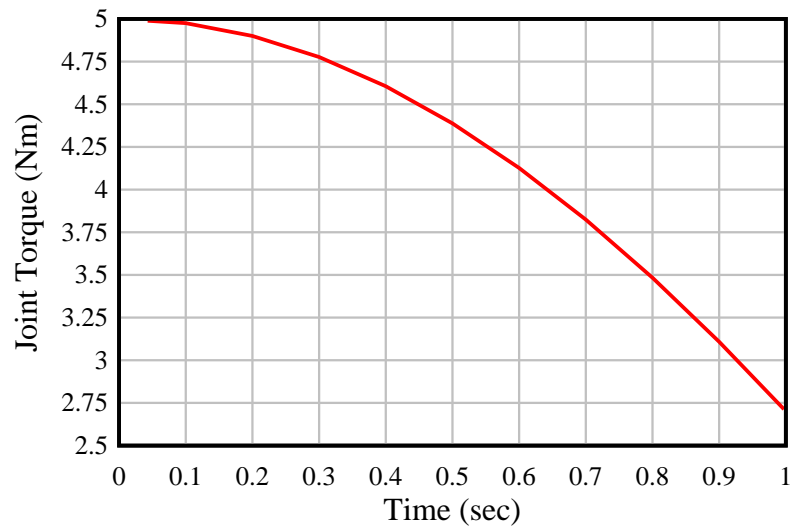


Figure 5 Rigid Link Joint Torque

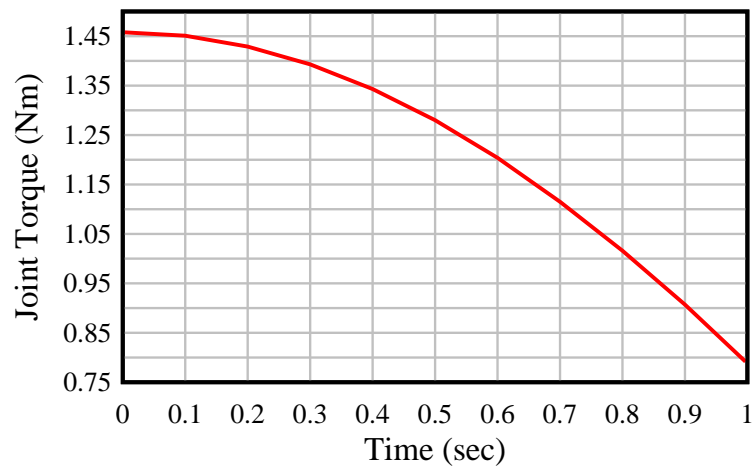


Figure 6 Joint Torque for flexible link

Figure 7 shows the difference between the joint angle and the tip angle for the flexible link as the tip angle and the hub angle varies because of the flexibility of the link considered.

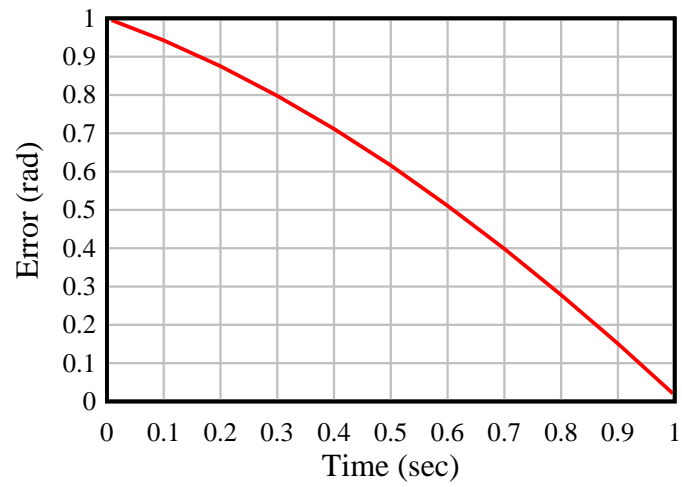


Figure 7 Error of Joint and Tip angle

Trajectory 2

The second trajectory considered with the following polar equation,

$$\theta = r \cos(2\pi t)$$

Where t is the time in seconds.

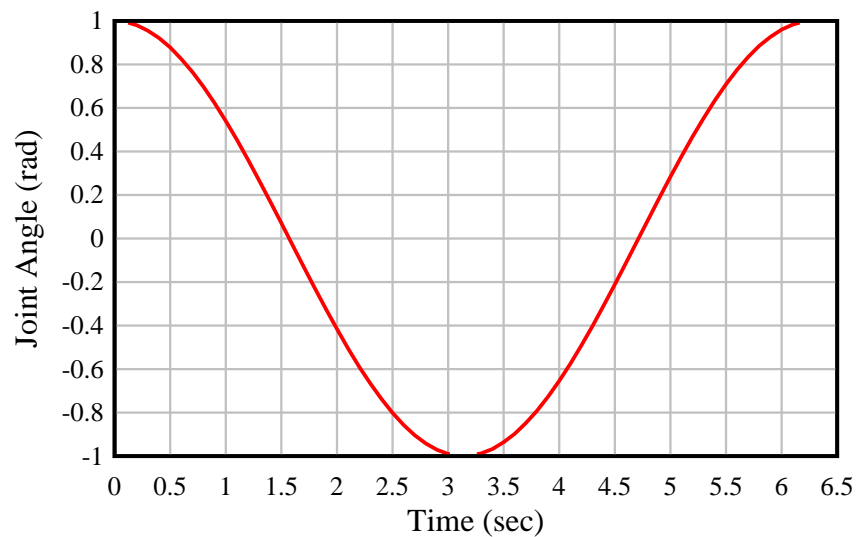


Figure 8 Trajectory 2

Using inverse dynamic analysis for the above trajectory the corresponding joint torques of rigid and flexible links are shown in the following figures 9 and 10 respectively. Here we can observe that the required joint torques of the rigid link are more than flexible link.

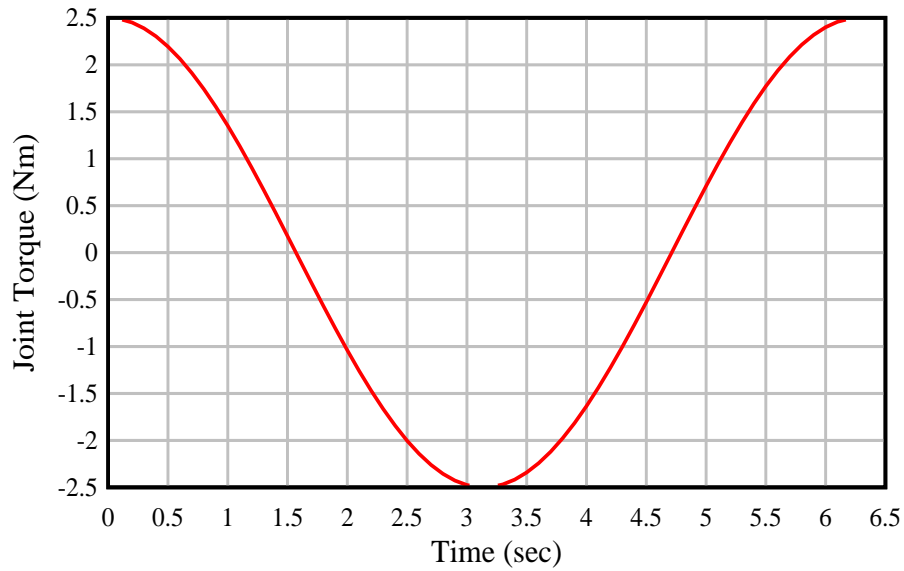


Figure 9 Torque for Rigid link in trajectory 2

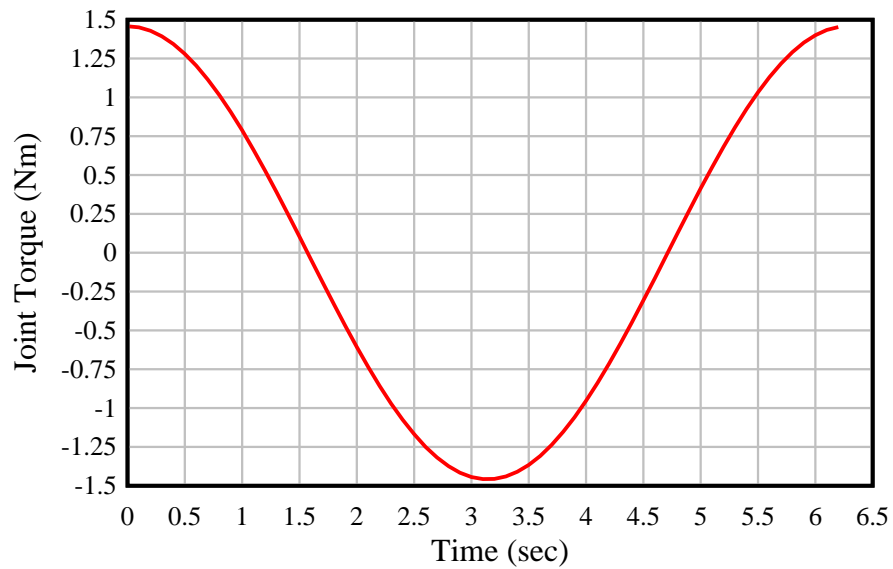


Figure 10 Torque for flexible link in trajectory 2

Figure 11 shows the difference between the joint angle and the tip angle for the flexible link in the second trajectory.

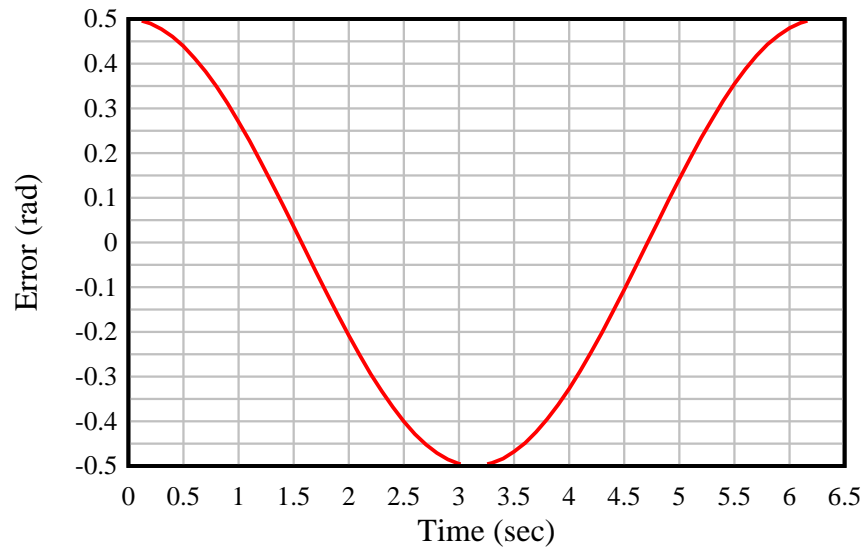


Figure 11 Error of tip and hub angle in trajectory 2

PD Results:

The PD control results of the rigid link on trajectory 1 are shown below. Here the proportional derivatives considered are $K_p = 100$, $K_d = 20$. For the best performance of the controller parameters the trajectory tracking of the end effector with the desired trajectory shown in the figure 12.

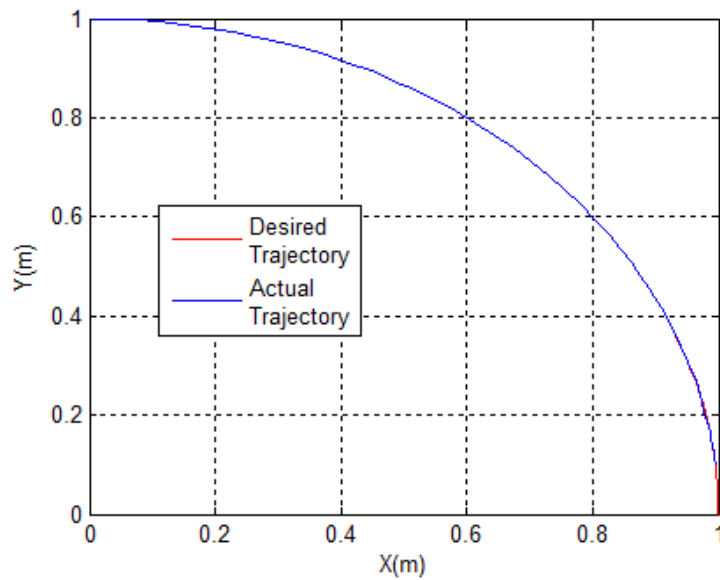


Figure 12 Desired and Actual Trajectory

For tracking the actual trajectory the controlled torques are plotted in figure 13.

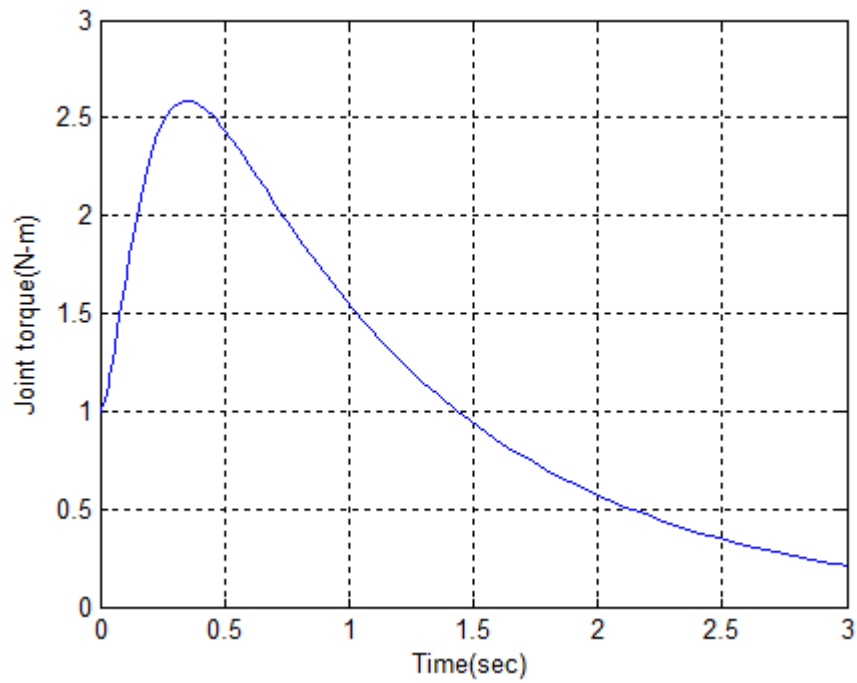


Figure 13 Joint Torques

Figure 14 shows the position errors of the end effector. Here the error reaches 0 asymptotically.

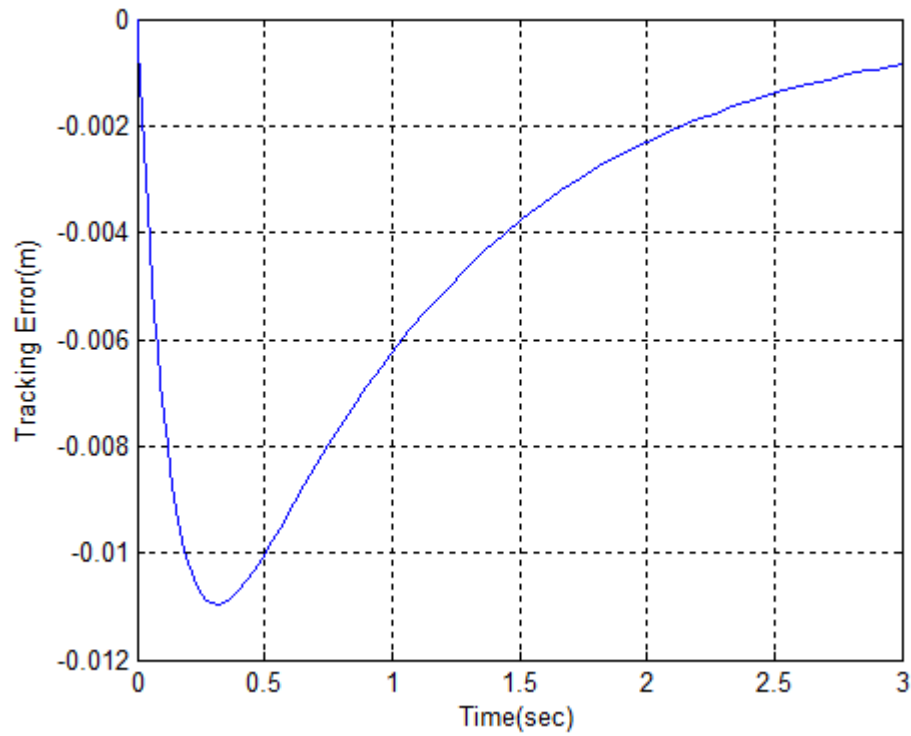


Figure 14 Position Errors of End Effector

The following figures 15 and 16 shows the variation of joint angle and velocities of the actual trajectory obtained by the PD controller.

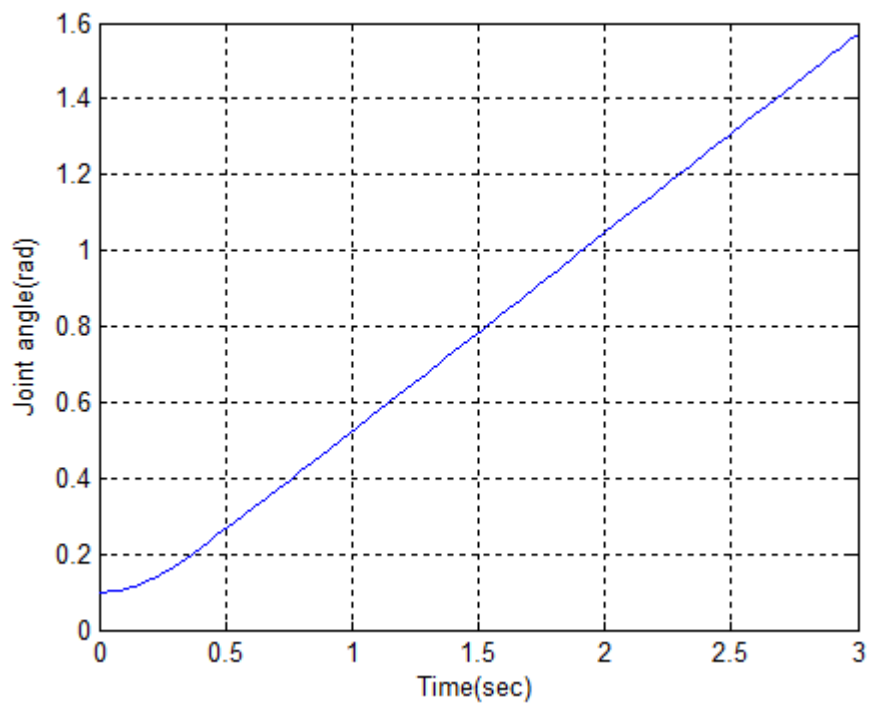


Figure 15 Joint Angles of Actual Trajectory

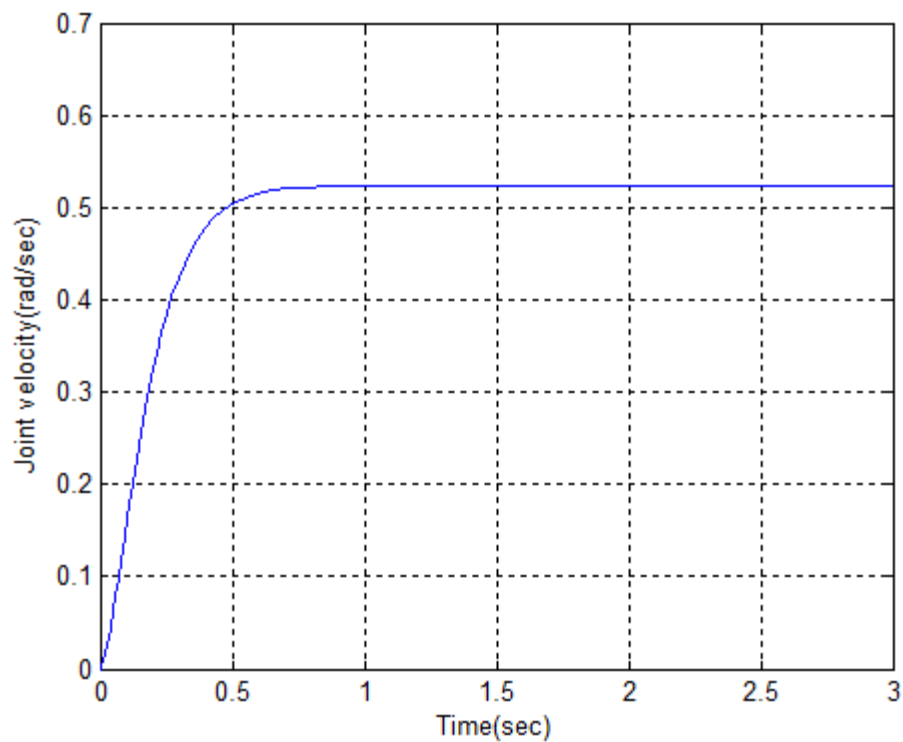


Figure 16 Joint Velocities of Actual Trajectory

CHAPTER 5: CONCLUSION

5.1 SUMMARY

The dynamics of the flexible link is studied and modelled in a MATLAB to differentiate between rigid and flexible links and their dynamic behaviors such as torque requirement, trajectory motion and the tip deflection, hub angles are identified. The torque required to move a flexible link in a given trajectory is found to be comparatively lesser than that of the torque requirement of a rigid link.

5.2 FUTURE SCOPE OF WORK

1. Hardware implementation of the proposed system can be done.
2. Further enhancement in the controller design by including more state/model – free mathematical algorithms such as GA, neural network.
3. The model is built by Euler – Bernoulli thin beam theory which is obviously not the most general model available which can be improved upon.
4. The FLC can be made more robust and optimized by training it beforehand with data set and optimizing with an algorithm such as DE and PSO.
5. Other controller designs such as PID based controller or H- controllers can be joined with the main controller to boost the performance of the damping controller.

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